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$$\phi'=\tfrac{1}{2}\theta, \text{ and from (1), } \mu=\frac{\sin\phi}{\sin\phi'}\ldots(9),$$

but from (8), $\phi=\frac{1}{2}(D+2\phi')=\frac{1}{2}(D+\theta)$. Hence, substituting in (9), we have,

$$\mu=\frac{\sin\frac{1}{2}(D+\theta)}{\sin\frac{1}{2}\theta}\ldots(10),$$

which gives the relation between the minimum deviation D, the angle of the prism θ, and the index of refraction μ.

Also solved by G. B. M. Zerr and J. Scheffer.

284. Proposed by L. H. McDONALD, M. A., Ph. D., Sometimes Tutor at Cambridge, Jersey City, N. J.

Inscribe the triangle of maximum area in a given circle.

Solution by PROFESSOR F. L. GRIFFIN, Williams College.

Let the diameter, of length d, through any vertex make angles α and β with the two sides of the triangle meeting there. Then two sides of the triangle are $d\cos\alpha$ and $d\cos\beta$; and their included angle is $\alpha+\beta$. [$\alpha-\beta$ is excluded, since a triangle contained in a semi-circle clearly can not be the maximum.] Now the area is $\frac{1}{2}(d\cos\alpha).(d\cos\beta).\sin(\alpha+\beta)$; so that we have to render a maximum the function $F(\alpha, \beta)\equiv\cos\alpha.\cos\beta.\sin(\alpha+\beta)$.

$$\text{But } \frac{\partial F}{\partial \alpha}=\cos\beta\,(\cos\alpha.\cos\overline{\alpha+\beta}-\sin\alpha.\sin\overline{\alpha+\beta})=\cos\beta.\cos(2\alpha+\beta),$$

$$\frac{\partial F}{\partial \beta}=\cos\alpha.\cos(2\beta+\alpha).$$

Equating these to zero, we obtain since $\cos\alpha\neq0$, $\cos\beta\neq0$ if there is actually a triangle:

$$2\alpha+\beta=\tfrac{1}{2}\pi,\quad 2\beta+\alpha=\tfrac{1}{2}\pi,\quad \text{whence } \alpha=\beta=\tfrac{1}{6}\pi.$$

$$\text{Also, } \frac{\partial^2 F}{\partial \alpha^2}=-2\sin(2\alpha+\beta)\cos\beta,\quad \frac{\partial^2 F}{\partial\alpha\,\partial\beta}=-\sin\overline{2\alpha+2\beta},$$

$$\frac{\partial^2 F}{\partial \beta^2}=-2\sin(2\beta+\alpha).\cos\alpha;\ \text{whence, } \left[\frac{\partial^2 F}{\partial \alpha^2}.\frac{\partial^2 F}{\partial \beta^2}-\left(\frac{\partial^2 F}{\partial\alpha\,\partial\beta}\right)^2\right]_{\alpha=\beta=\frac{1}{6}\pi}=\tfrac{3}{2},$$

the positive result together with the negative value of $\partial^2 F/\partial\alpha^2$ showing a maximum value of $F(\alpha, \beta)$.

For $\alpha=\beta=\frac{1}{6}\pi$, the sides $d\cos\alpha$, $d\cos\beta$ are equal, and the isosceles triangle is equilateral, having its vertex angle equal to $\frac{1}{3}\pi$.

Also solved by H. C. Feemster, Wilmer Thompson, G. B. M. Zerr, and C. N. Schmall.

285. Proposed by C. N. SCHMALL, 604 East 5th Street, New York City.

If R_1 and R_2 are the radii of curvature of an ellipse at the extremities of a pair of conjugate diameters, show that $R_1^{2/3}+R_2^{2/3}=\frac{a^2+b^2}{(ab)^{2/3}}$, where a, b, are the semi-axes.

I. Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Let $x^2/a^2+y^2/b^2=1$ be the ellipse. Also let $x=a\cos\theta$, $y=b\sin\theta$.

$$dx=-a\sin\theta\,d\theta,\quad d^2x=-a\cos\theta\,d\theta^2-a\sin\theta\,d^2\theta,$$
$$dy=b\cos\theta\,d\theta,\quad d^2y=-b\sin\theta\,d\theta^2+b\cos\theta\,d^2\theta.$$

$$\therefore R_1=\frac{(dx^2+dy^2)^{\frac{3}{2}}}{dxd^2y-dy\,d^2x}=\frac{(a^2\sin^2\theta+b^2\cos^2\theta)^{\frac{3}{2}}}{ab}.$$

$$R_2=\frac{(a^2\sin^2\phi+b^2\cos^2\phi)^{\frac{3}{2}}}{ab}.\quad\text{But }\phi=\theta+\tfrac{1}{2}\pi.$$

$$\therefore R_2=\frac{(a^2\cos^2\theta+b^2\sin^2\theta)^{\frac{3}{2}}}{ab}.$$

$$R_1^{2/3}+R_2^{2/3}=\frac{(a^2+b^2)\,(\sin^2\theta+\cos^2\theta)}{(ab)^{2/3}}=\frac{a^2+b^2}{(ab)^{2/3}}.$$

II. Solution by HOWARD C. FEEMSTER, Professor of Mathematics, York College, York, Neb.; S. G. BARTON, Professor of Mathematics, Clarkson School of Technology, Potsdam, N. Y., and J. SCHEFFER, Hagerstown, Md.

Let x_1, y_1 be the point of intersection of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, with a diameter, and $-\frac{ay_1}{b}$, $\frac{bx_1}{a}$, the point of intersection of the ellipse with the diameter conjugate to the first.

But the radius of curvature of any point on a curve equals:

$$(1)\quad \rho=\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}.$$